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Observable contributions of new exotic quarks to quark mixing

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ABSTRACT: Models with new vector-like quarks can produce observable quark mixing effects which are forbidden in the Standard Model. We classify all such models and write down the effective Lagrangian that results from integrating out the new quarks. We study the relations between neutral and charged currents and discuss how to distinguish among the different possibilities.

Contents

1. Introduction

In a recent paper [1] we use an effective Lagrangian approach (see [2, 3, 4] and references there in) to study the mixing of different flavours of quarks in trilinear couplings for general extensions of the Standard Model (SM). There we show that, under quite general assumptions, only models containing new exotic quarks near the TeV scale can produce large non-standard effects in the vertices $V\bar{q}q'$ and $H\bar{q}q'$, where V is a massive gauge boson and H , the Higgs boson. In this paper we study in detail the mixing of the known quarks in SM extensions that include any addition of vector-like quarks. Our method will be to integrate out the new heavy fermions to obtain the corresponding effective Lagrangian. The advantage of this approach is that it isolates the interesting physics at the accessible energies and allows us to discuss the different models with an arbitrary number of exotic quarks in a unified framework. Furthermore, we can use directly the results in Ref. [1]. Many aspects of quark mixing, mainly in extensions with new vector-like singlet and doublet quarks, have been studied in the literature [5, 6]. Here we find new relations for these models and consider isotriplets in detail in this context for the first time.

Let us first review the general effective description of [1] and the arguments for the relevance of vector-like quarks. Based on the success of precision tests of the SM, we assume that the only light fields are the ones in the SM, with three families of quarks and leptons and one Higgs doublet. Physics up to a scale Λ is described by the effective Lagrangian

$$\mathcal{L}^{eff} = \mathcal{L}_4 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \dots \quad (1.1)$$

\mathcal{L}_4 is the SM Lagrangian and \mathcal{L}_6 describes corrections arising from new physics at a scale Λ . The latter can be written as

$$\mathcal{L}_6 = \alpha_x \mathcal{O}_x + \text{h.c.}, \quad (1.2)$$

where \mathcal{O}_x is a gauge invariant operator of dimension 6 and we use the repeated index convention unless otherwise specified. The entire \mathcal{L}^{eff} must be invariant under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry of the SM. No $1/\Lambda$ terms are included in Eq. (1.1) because when the almost exact lepton and baryon number conservation is also imposed, dimension 5 operators involving the SM fields are forbidden [3]. The equations of motion of \mathcal{L}_4 allow to reduce the number of operators \mathcal{O}_x to 80, up to flavour indices.

\mathcal{L}^{eff} parametrises a large class of SM extensions, which includes renormalizable gauge models with extra fermions, scalars and gauge bosons, supersymmetric [7] or not and in four or higher dimensions. The operators \mathcal{O}_x are generated by the exchange of the heavy degrees of freedom in the full theory. It is useful to distinguish those operators which are generated at tree level from the ones which are only generated by loop diagrams [4]. If the high-energy theory is weak interacting, which we assume henceforth, the coefficients corresponding to loop-generated operators have an additional $1/16\pi^2$ suppression. Therefore, large effects require that the new physics contributes at tree level. Here we are mainly interested in the vertices $V\bar{q}q'$ and $H\bar{q}q'$. Processes with one of these bosons in the final state give direct information about these two-quark couplings. The fact that four-fermion operators (of dimension 6) do not contribute to these processes greatly reduces the number of operators to be considered: only 7 tree-level generated operators are relevant [4]. These operators are listed in Ref. [1].

After spontaneous symmetry breaking (SSB), the 7 operators give corrections to \mathcal{L}_4 of order $\alpha_x \frac{v^2}{\Lambda^2}$, with $v \sim 250$ GeV the electroweak vacuum expectation value (vev). If the high energy theory is a renormalizable gauge theory, the operators that modify trilinear quark-gauge couplings can only be generated at tree level by either extra gauge bosons mixing with the Z or W^\pm or extra quarks mixing through Yukawa couplings with the SM ones. However, the LEP data indicate that the mixing θ between SM and heavy gauge bosons is $\lesssim 0.01$ [8, 9], which gives an additional suppression. Hence we are led to the conclusion that only extra quarks can induce large new effects in quark mixing.

A fourth generation of chiral fermions is excluded at 99% C.L. by the present limits on the S parameter [9]. On the other hand, vector-like fermions of mass larger than the mass of the top are allowed. In order to contribute to quark mixing, the new fermions must couple to the SM quarks (through Yukawa couplings). This requires that they transform as quarks under $SU(3)_C$. There are several possibilities for their electroweak quantum numbers. In Table 1 we catalogue the exotic quarks Q which can mix with the SM fermions through a Yukawa coupling to the SM scalar [6]. Sometimes we shall refer to each of these types of vector-like quarks as $Q^{(m)}$, where $m = 1, \dots, 7$ indicates its position in Table 1. We stick to the SM scalar sector because scalar singlets do not mix inequivalent representations and scalar triplets cannot have the large vevs necessary to induce observable quark mixing, while if there are additional scalar doublets we can consider the combination getting a vev. Note that only one chirality of each type of vector-like quark couples in this way to the SM fermions. In the following we consider the addition of any number of these exotic fermions to the SM. We integrate them out to obtain the corresponding effective Lagrangian. Then we use the results in Ref. [1] to write down the corrections to the SM gauge and Yukawa couplings and find the relations and bounds satisfied by the couplings in each model, which may allow to discriminate between the different possibilities.

Table 1: Vector-like quark multiplets $Q^{(m)}$ mixing with the SM quarks through Yukawa couplings. The index m labels the seven types of quark multiplet additions in the given order. The electric charge is the sum of the third component of isospin T_3 and the hypercharge Y .

$Q^{(m)}$	U	D	$\begin{pmatrix} U \\ D \end{pmatrix}$	$\begin{pmatrix} X \\ U \end{pmatrix}$	$\begin{pmatrix} D \\ Y \end{pmatrix}$	$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$
isospin	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
hypercharge	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{7}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$

2. Effective Lagrangian for extensions with exotic quarks

The complete Lagrangian for the quark sector is

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_h + \mathcal{L}_{lh}, \quad (2.1)$$

where \mathcal{L}_l and \mathcal{L}_h involve only light and heavy fields, respectively, and \mathcal{L}_{lh} contains their mixing. \mathcal{L}_l is the SM Lagrangian and coincides with \mathcal{L}_4 . Its quark sector reads

$$\mathcal{L}_l^{\text{quark}} = \bar{q}_L^i i \not{D} q_L^i + \bar{u}_R^i i \not{D} u_R^i + \bar{d}_R^i i \not{D} d_R^i - \left(V_{ij}^\dagger \lambda_j^u \bar{q}_L^i \tilde{\phi} u_R^j + \lambda_i^d \bar{q}_L^i \phi d_R^i + \text{h.c.} \right), \quad (2.2)$$

with i, j running from 1 to 3. We assume without loss of generality that the down Yukawa couplings λ_i^d are diagonal. The up Yukawas can be diagonalized after SSB by a rotation of u_L , producing the Cabibbo-Kobayashi-Maskawa (CKM) matrix V in the charged currents. The scalar doublet has hypercharge $Y = 1/2$ (ϕ) or $Y = -1/2$ ($\tilde{\phi} = \epsilon \phi^*$, with ϵ the antisymmetric tensor of rank 2). \mathcal{L}_h is quadratic in the heavy quarks:

$$\mathcal{L}_h = \bar{Q}_L^a i \not{D} Q_L^a + \bar{Q}_R^a i \not{D} Q_R^a - M_a (\bar{Q}_L^a Q_R^a + \bar{Q}_R^a Q_L^a) - (\tilde{\lambda}_{ab} \bar{Q}_L^a \Phi_{ab} Q_R^b + \text{h.c.}), \quad (2.3)$$

with a, b running from 1 to an arbitrary n . Q^a can be any multiplet in Table 1. For a given type of multiplet, a and b will also label the different copies. Φ_{ab} represents the appropriate form of the scalar required by gauge invariance. The different possibilities are displayed in Table 2. The mass terms in Eq. (2.3) can be assumed to be diagonal and real without loss of generality. These masses, which are not protected by the gauge symmetry, determine the scale in the effective Lagrangian: $\Lambda \sim M_a > v$. On the other hand, the Yukawa couplings for the heavy quarks are non-diagonal and of order 1. Finally, \mathcal{L}_{lh} contains the Yukawa couplings mixing SM and new vector-like quarks. In Table 3 we collect these terms for each type of multiplet in Table 1. We use primes for these Yukawa couplings and include V in the definition of some of them in order to simplify the final expressions. As we pointed out only one chirality of each type of vector-like multiplet mixes. Because \mathcal{L}_{lh} is linear in heavy fields, it can be written

$$\mathcal{L}_{lh} = \bar{Q}_L^a \frac{\delta \mathcal{L}_{lh}}{\delta \bar{Q}_L^a} + \bar{Q}_R^a \frac{\delta \mathcal{L}_{lh}}{\delta \bar{Q}_R^a} + \text{h.c.}, \quad (2.4)$$

Table 2: Values of Φ_{ab} for the different possibilities of mixing between heavy fields.

	U_R^b	D_R^b	$\begin{pmatrix} X \\ U \\ D \end{pmatrix}_R^b$	$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}_R^b$
$\overline{\begin{pmatrix} U \\ D \end{pmatrix}_L^a}$	$\tilde{\phi}$	ϕ	$\frac{\sigma^I}{2}\tilde{\phi}$	$\frac{\sigma^I}{2}\phi$
$\overline{\begin{pmatrix} X \\ U \end{pmatrix}_L^a}$	ϕ	—	$\frac{\sigma^I}{2}\phi$	—
$\overline{\begin{pmatrix} D \\ Y \end{pmatrix}_L^a}$	—	$\tilde{\phi}$	—	$\frac{\sigma^I}{2}\tilde{\phi}$

where $\frac{\delta\mathcal{L}_{lh}}{\delta Q^a}$ can be read from Table 3.

In order to find the effective Lagrangian describing the physics below the scale Λ , we integrate out the heavy modes for a generic addition of vector-like multiplets. Note that, unlike chiral fermions [11], vector-like quarks decouple when their mass is sent to infinity. For our purposes it is sufficient to perform the integration at tree level, which can be carried out imposing the equations of motion of the heavy modes. There is no need to consider each kind of exotic quark separately, as \mathcal{L}_{lh} has essentially the same form in all cases (see Ref. [1], however, for a particular example). The requirement that the action be stationary under variations of the heavy fields $\bar{Q}_{L,R}^a$ gives two coupled equations of motion (with no sum in a):

$$i\not{D}Q_L^a - M_a Q_R^a - \tilde{\lambda}_{ab}\Phi_{ab}Q_R^b + \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^a} = 0, \quad (2.5)$$

$$i\not{D}Q_R^a - M_a Q_L^a - \tilde{\lambda}_{ab}^\dagger\Phi_{ab}^\dagger Q_L^b + \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} = 0, \quad (2.6)$$

where we have used Eqs. (2.3) and (2.4). These equations can be solved iteratively. The solution to order $1/M^2$ is

$$Q_R^a = \frac{i\not{D}}{M_a^2} \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} + \frac{1}{M_a} \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^a} - \frac{1}{M_a M_b} \tilde{\lambda}_{ab}\Phi_{ab} \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^b}, \quad (2.7)$$

$$Q_L^a = \frac{i\not{D}}{M_a^2} \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^a} + \frac{1}{M_a} \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} - \frac{1}{M_a M_b} \tilde{\lambda}_{ab}^\dagger\Phi_{ab}^\dagger \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^b}. \quad (2.8)$$

Inserting these expressions in the original Lagrangian in Eq.(2.1) we obtain, to order $1/M^2$,

$$\mathcal{L}_4 = \mathcal{L}_l, \quad (2.9)$$

$$\frac{1}{\Lambda^2}\mathcal{L}_6 = \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^a} \frac{i\not{D}}{M_a^2} \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^a} + \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} \frac{i\not{D}}{M_a^2} \frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} - \left(\frac{\delta\mathcal{L}_{lh}}{\delta Q_R^a} \frac{1}{M_a M_b} \tilde{\lambda}_{ab}\Phi_{ab} \frac{\delta\mathcal{L}_{lh}}{\delta Q_L^b} + \text{h.c.} \right). \quad (2.10)$$

Table 3: Yukawa terms mixing light (q_L^i, u_R^i, d_R^i) and heavy $(Q_{L,R}^a)$ quarks. The hermitian conjugate terms must be added. The index I in the Pauli matrices corresponds to the $(+, 0, -)$ basis of isospin, as it does in the vector-like triplets. The superscript $m = 1, \dots, 7$ in $\lambda'^{(m)}$ labels the different type of multiplet addition. $\frac{\delta \mathcal{L}_{lh}}{\delta Q_{L,R}^a}$ can be read directly from \mathcal{L}_{lh} which is linear in $\bar{Q}_{L,R}^a$.

$Q^{(m)}$	$-\mathcal{L}_{lh}$
U	$\lambda'_{aj(1)} V_{ji} \bar{U}_R^a \tilde{\phi}^\dagger q_L^i$
D	$\lambda'_{ai(2)} \bar{D}_R^a \phi^\dagger q_L^i$
$\begin{pmatrix} U \\ D \end{pmatrix}$	$\lambda'_{ai(3u)} \overline{\begin{pmatrix} U \\ D \end{pmatrix}_L^a} \tilde{\phi} u_R^i + \lambda'_{ai(3d)} \overline{\begin{pmatrix} U \\ D \end{pmatrix}_L^a} \phi d_R^i$
$\begin{pmatrix} X \\ U \end{pmatrix}$	$\lambda'_{ai(4)} \overline{\begin{pmatrix} X \\ U \end{pmatrix}_L^a} \phi u_R^i$
$\begin{pmatrix} D \\ Y \end{pmatrix}$	$\lambda'_{ai(5)} \overline{\begin{pmatrix} D \\ Y \end{pmatrix}_L^a} \tilde{\phi} d_R^i$
$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\lambda'_{aj(6)} V_{ji} \overline{\begin{pmatrix} X \\ U \\ D \end{pmatrix}_{RI}^a} \tilde{\phi}^\dagger \frac{\sigma^I}{2} q_L^i$
$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$	$\lambda'_{aj(7)} V_{ji} \overline{\begin{pmatrix} U \\ D \\ Y \end{pmatrix}_{RI}^a} \phi^\dagger \frac{\sigma^I}{2} q_L^i$

We have set $\frac{\delta \mathcal{L}_{lh}}{\delta Q_L^a} \frac{\delta \mathcal{L}_{lh}}{\delta Q_R^a} = 0$ since for each a one of the two factors vanishes. Diagrammatically these dimension 6 corrections follow from the Feynman diagrams in Fig. 1. The addition of the first two diagrams generates the operators with a covariant derivative, whereas the last diagram generates the operators in parentheses in Eq. (2.10).

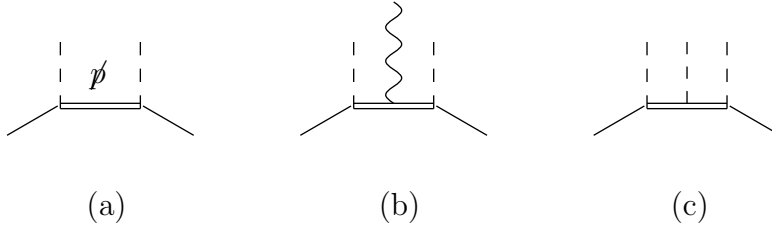


Figure 1: Diagrams contributing to dimension 6 operators in the effective Lagrangian. Light (heavy) quarks are depicted by a single (double) line.

In order to use the results of Ref. [1] we have to write \mathcal{L}_6 in terms of the operators of

Ref. [3]. The covariant derivative in Eq.(2.10) acts on $\frac{\delta \mathcal{L}_\mu}{\delta Q^a}$, which is the product of the scalar and a light quark multiplet (see Table 3). Using the (covariant) Leibniz rule we obtain terms in which the covariant derivative acts either on the scalar field or on the light quark. Whereas the former give—after a simple Fierz reordering of the representation indices—operators in the catalogue of [3], the latter have to be transformed using the equations of motion of \mathcal{L}_4 . The result takes the form (see Ref. [3] for notation)

$$\begin{aligned} \mathcal{L}_6 = & (\alpha_{\phi q}^{(1)})_{ij}(\phi^\dagger i D_\mu \phi)(\bar{q}_L^i \gamma^\mu q_L^j) + (\alpha_{\phi q}^{(3)})_{ij}(\phi^\dagger \sigma^I i D_\mu \phi)(\bar{q}_L^i \gamma^\mu \sigma^I q_L^j) \\ & + (\alpha_{\phi u})_{ij}(\phi^\dagger i D_\mu \phi)(\bar{u}_R^i \gamma^\mu u_R^j) + (\alpha_{\phi d})_{ij}(\phi^\dagger i D_\mu \phi)(\bar{d}_R^i \gamma^\mu d_R^j) \\ & + (\alpha_{\phi \phi})_{ij}(\phi^T \epsilon i D_\mu \phi)(\bar{u}_R^i \gamma^\mu d_R^j) + (\alpha_{u\phi})_{ij}(\phi^\dagger \phi)(\bar{q}_L^i \tilde{\phi} u_R^j) \\ & + (\alpha_{d\phi})_{ij}(\phi^\dagger \phi)(\bar{q}_L^i \phi d_R^j) + \text{h.c.} \quad . \end{aligned} \quad (2.11)$$

Note that vector-like quarks generate only these 7 operators. In particular, operators of magnetic-moment type or with stress-energy tensors do not appear at this order. Four-fermion operators are not generated either, although one should keep in mind that they may arise from other kinds of new physics. Each vector-like quark gives an independent contribution to one of the first two terms in Eq. (2.10), which is diagonal in the heavy quark flavour, a . The term in parentheses in this equation—which contributes only to $\mathcal{O}_{u\phi}$ and $\mathcal{O}_{d\phi}$ —requires the cooperative participation of two different types of vector-like multiplets. Thus, the coefficients in Eq. (2.11) can be written

$$\alpha_x = \sum_m \alpha_x^m + \sum_{m < n} \alpha_x^{mn}, \quad (2.12)$$

where α_x^m is the diagonal contribution of the vector-like multiplets $Q^{(m)}$ and α_x^{mn} is the combined contribution of multiplets $Q^{(m)}$ and $Q^{(n)}$. Note that $\alpha_x^{mn} = 0$ unless $x = u\phi, d\phi$, and only one of the two quarks is a doublet. The contributions α_x^m and α_x^{mn} are collected in Table 4 and Table 5, respectively.

3. Corrections to quark couplings

Upon SSB, the 7 operators in \mathcal{L}_6 give $\frac{v^2}{\Lambda^2}$ corrections to the SM Lagrangian. In the mass eigenstate basis these corrections read:

$$\begin{aligned} \mathcal{L}^Z = & -\frac{g}{2 \cos \theta_W} \left(\bar{u}_L^i X_{ij}^{uL} \gamma^\mu u_L^j + \bar{u}_R^i X_{ij}^{uR} \gamma^\mu u_R^j \right. \\ & \left. - \bar{d}_L^i X_{ij}^{dL} \gamma^\mu d_L^j - \bar{d}_R^i X_{ij}^{dR} \gamma^\mu d_R^j - 2 \sin^2 \theta_W J_{EM}^\mu \right) Z_\mu, \\ \mathcal{L}^W = & -\frac{g}{\sqrt{2}} (\bar{u}_L^i W_{ij}^L \gamma^\mu d_L^j + \bar{u}_R^i W_{ij}^R \gamma^\mu d_R^j) W_\mu^+ + \text{h.c.}, \\ \mathcal{L}^H = & -\frac{1}{\sqrt{2}} (\bar{u}_L^i Y_{ij}^u u_R^j + \bar{d}_L^i Y_{ij}^d d_R^j) H + \text{h.c.}, \end{aligned} \quad (3.1)$$

with

$$X_{ij}^{uL} = \delta_{ij} - \frac{v^2}{\Lambda^2} V_{ik} (\alpha_{\phi q}^{(1)} - \alpha_{\phi q}^{(3)})_{kl} V_{lj}^\dagger,$$

$$\begin{aligned}
X_{ij}^{uR} &= -\frac{v^2}{\Lambda^2}(\alpha_{\phi u})_{ij}, \\
X_{ij}^{dL} &= \delta_{ij} + \frac{v^2}{\Lambda^2}(\alpha_{\phi q}^{(1)} + \alpha_{\phi q}^{(3)})_{ij}, \\
X_{ij}^{dR} &= \frac{v^2}{\Lambda^2}(\alpha_{\phi d})_{ij}, \\
W_{ij}^L &= \tilde{V}_{ik}(\delta_{kj} + \frac{v^2}{\Lambda^2}(\alpha_{\phi q}^{(3)})_{kj}), \\
W_{ij}^R &= -\frac{1}{2}\frac{v^2}{\Lambda^2}(\alpha_{\phi\phi})_{ij}, \\
Y_{ij}^u &= \delta_{ij}\lambda_j^u - \frac{v^2}{\Lambda^2}\left(V_{ik}(\alpha_{u\phi})_{kj} + \frac{1}{4}\delta_{ij}[V_{ik}(\alpha_{u\phi})_{kj} + (\alpha_{u\phi})_{ik}^\dagger V_{kj}^\dagger]\right), \\
Y_{ij}^d &= \delta_{ij}\lambda_j^d - \frac{v^2}{\Lambda^2}\left((\alpha_{d\phi})_{ij} + \frac{1}{4}\delta_{ij}(\alpha_{d\phi} + \alpha_{d\phi}^\dagger)_{ij}\right),
\end{aligned} \tag{3.2}$$

where we have introduced the unitary matrix

$$\tilde{V} = V + \frac{v^2}{\Lambda^2}(VA_L^d - A_L^u V). \tag{3.3}$$

$A_L^{u,d}$ are the antihermitian matrices which, together with $A_R^{u,d}$, diagonalize the corrected mass terms:

$$(A_L^u)_{ij} = \frac{1}{2}(1 - \frac{1}{2}\delta_{ij})\frac{\lambda_i^u(V\alpha_{u\phi})_{ij}^\dagger + (-1)^{\delta_{ij}}(V\alpha_{u\phi})_{ij}\lambda_j^u}{(\lambda_i^u)^2 - (-1)^{\delta_{ij}}(\lambda_j^u)^2}, \tag{3.4}$$

$$(A_L^d)_{ij} = \frac{1}{2}(1 - \frac{1}{2}\delta_{ij})\frac{\lambda_i^d(\alpha_{d\phi})_{ij}^\dagger + (-1)^{\delta_{ij}}(\alpha_{d\phi})_{ij}\lambda_j^d}{(\lambda_i^d)^2 - (-1)^{\delta_{ij}}(\lambda_j^d)^2}. \tag{3.5}$$

\mathcal{L}_6 in Eq. (2.11) does not generate any other trilinear coupling. A possible derivative coupling to the Higgs is forbidden in these models because the corresponding coefficients α_x are always hermitian (see Ref. [1]). With Eq. (3.2) and Tables 4 and 5 we can answer phenomenological questions on quark mixing in processes with a vector boson or a Higgs.

4. Relations and Bounds

As can be readily observed from Eq. (3.2), this effective Lagrangian gives mixing effects which are forbidden in the SM. In general:

- There are flavour changing neutral currents (FCNC) in the gauge interactions at tree level, as the GIM mechanism [12] does not apply: $X_{ij}^{uL}, X_{ij}^{dL} \neq \delta_{ij}$.
- There are right-handed (RH) neutral currents not proportional to J_{EM}^μ : $X_{ij}^{uR}, X_{ij}^{dR} \neq 0$.
- The left-handed (LH) charged currents are not described by a unitary CKM matrix: $W_{ik}^L W_{kj}^{L\dagger}, W_{ik}^{L\dagger} W_{kj}^L \neq \delta_{ij}$.

Table 4: Coefficients α_x^m resulting from the integration of an arbitrary number of each type of vector-like quarks. The superscript $m = 1, \dots, 7$ in $\lambda'^{(m)}$ labels the different type of multiplet addition.

$Q^{(m)}$	$\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi q}^{(3)})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi u})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi \phi})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{u\phi})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{d\phi})_{ij}}{\Lambda^2}$
U	$\frac{1}{4} V_{ik}^\dagger \frac{\lambda'^{(1)\dagger} \lambda'^{(1)}}{M_a^2} V_{lj}$	$-\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$2 \frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2} V_{kj}^\dagger \lambda_j^u$	—
D	$-\frac{1}{4} \frac{\lambda'^{(2)\dagger} \lambda'^{(2)}}{M_a^2}$	$\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	—	$-2 \frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2} \lambda_j^d$
$\begin{pmatrix} U \\ D \end{pmatrix}$	—	—	$-\frac{1}{2} \frac{\lambda'^{(3u)\dagger} \lambda'^{(3u)}}{M_a^2}$	$\frac{1}{2} \frac{\lambda'^{(3d)\dagger} \lambda'^{(3d)}}{M_a^2}$	$-\frac{\lambda'^{(3u)\dagger} \lambda'^{(3d)}}{M_a^2}$	$-V_{ik}^\dagger \lambda_k^u \frac{(\alpha_{\phi u})_{kj}}{\Lambda^2}$	$\lambda_i^d \frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$
$\begin{pmatrix} X \\ U \end{pmatrix}$	—	—	$\frac{1}{2} \frac{\lambda'^{(4)\dagger} \lambda'^{(4)}}{M_a^2}$	—	—	$V_{ik}^\dagger \lambda_k^u \frac{(\alpha_{\phi u})_{kj}}{\Lambda^2}$	—
$\begin{pmatrix} D \\ Y \end{pmatrix}$	—	—	—	$-\frac{1}{2} \frac{\lambda'^{(5)\dagger} \lambda'^{(5)}}{M_a^2}$	—	—	$-\lambda_i^d \frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$
$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\frac{3}{16} V_{ik}^\dagger \frac{\lambda'^{(6)\dagger} \lambda'^{(6)}}{M_a^2} V_{lj}$	$\frac{1}{3} \frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$\frac{2}{3} \frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2} V_{kj}^\dagger \lambda_j^u$	$\frac{4}{3} \frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2} \lambda_j^d$
$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$	$-\frac{3}{16} V_{ik}^\dagger \frac{\lambda'^{(7)\dagger} \lambda'^{(7)}}{M_a^2} V_{lj}$	$-\frac{1}{3} \frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$-\frac{4}{3} \frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2} V_{kj}^\dagger \lambda_j^u$	$-\frac{2}{3} \frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2} \lambda_j^d$

Table 5: Coefficients α_x^{mn} resulting from the integration of an arbitrary number of vector-like quarks of each type due to the mixing between vector-like multiplets. The superscript $m = 1, \dots, 7$ in $\lambda'^{(m)}$ labels the different type of multiplet addition.

$Q^{(m)}, Q^{(n)}$	$\frac{(\alpha_{u\phi}^{mn})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{d\phi}^{mn})_{ij}}{\Lambda^2}$
$U, \begin{pmatrix} U \\ D \end{pmatrix}$	$V_{ik}^\dagger \frac{\lambda_{ka}'^{(1)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3u)}}{M_a M_b}$	—
$U, \begin{pmatrix} X \\ U \end{pmatrix}$	$V_{ik}^\dagger \frac{\lambda_{ka}'^{(1)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(4)}}{M_a M_b}$	—
$D, \begin{pmatrix} U \\ D \end{pmatrix}$	—	$\frac{\lambda_{ia}'^{(2)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3d)}}{M_a M_b}$
$D, \begin{pmatrix} D \\ Y \end{pmatrix}$	—	$\frac{\lambda_{ia}'^{(2)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(5)}}{M_a M_b}$
$\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\frac{1}{4} V_{ik}^\dagger \frac{\lambda_{ka}'^{(6)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3u)}}{M_a M_b}$	$\frac{1}{2} V_{ik}^\dagger \frac{\lambda_{ka}'^{(6)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3d)}}{M_a M_b}$
$\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} U \\ D \\ Y \end{pmatrix}$	$\frac{1}{2} V_{ik}^\dagger \frac{\lambda_{ka}'^{(7)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3u)}}{M_a M_b}$	$\frac{1}{4} V_{ik}^\dagger \frac{\lambda_{ka}'^{(7)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(3d)}}{M_a M_b}$
$\begin{pmatrix} X \\ U \end{pmatrix}, \begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$-\frac{1}{4} V_{ik}^\dagger \frac{\lambda_{ka}'^{(6)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(4)}}{M_a M_b}$	—
$\begin{pmatrix} D \\ Y \end{pmatrix}, \begin{pmatrix} U \\ D \\ Y \end{pmatrix}$	—	$-\frac{1}{4} V_{ik}^\dagger \frac{\lambda_{ka}'^{(7)\dagger} \tilde{\lambda}_{ab} \lambda_{bj}'^{(5)}}{M_a M_b}$

- There are RH charged currents: $W_{ij}^R \neq 0$
- There are FCNC mediated by the Higgs boson: $Y_{ij}^{u,d} \neq \sqrt{2} \delta_{ij} \frac{m_j^{u,d}}{v}$.

At this point it is important to remember that, whereas the couplings among the five lightest quarks are known with good precision, the couplings involving the top are only known with a precision of tens per cent. These couplings will be measured at the 1% level at future colliders [13], which sets the limit on new observable effects in top mixing. From our arguments above on large mixing effects it is then clear that only vector-like quarks (with $M/\lambda' \lesssim 1$ TeV) can induce observable new mixing effects in processes involving the top where a gauge boson or the Higgs is detected. Such effects would be described by Eq. (3.2) and Tables 4 and 5. Conversely, these processes can only prove this kind of new physics [1].

On the other hand, the couplings of the five lightest quarks are more constrained by present data and cannot be too large ($|X_{uc}| \lesssim 10^{-3}$ and $|X_{ds}| \lesssim 4 \times 10^{-5}$ for instance [6]).

Observable corrections to the couplings of the five lightest quarks can also arise from other kinds of new physics although, if present at the TeV scale, extra quarks would give the leading contribution for non-negligible λ' ($\lambda' \gtrsim \lambda_b$). For these small effects one must also take into account radiative corrections of similar size [14]. Typically, the most sensitive measurements of these couplings involve mixing of neutral mesons. One should be cautious in using our results in those processes, as four-quark operators can contribute as well. These operators are not generated by extra quarks but may be induced by other kinds of new physics, like extra gauge bosons. Even if four-quark operators are present, the results here may be employed to put bounds on the extra quark parameters assuming the absence of strong cancelations between cubic and quartic couplings. We observe that large new effects in top mixing and the required small new effects in the mixings of the other quarks can occur simultaneously in a natural way if the mechanism responsible for the hierarchy of the SM fermion masses produces a similar hierarchy in the Yukawa couplings mixing light and heavy quarks.

Once the couplings X_{ij} , W_{ij} [15] and Y_{ij} [16] have been measured (see Ref. [13] for a review), one may wonder what information they give about new physics above the electroweak scale. We have already argued that the observation of non-standard effects, especially in the case of top mixing, may point to the existence of new vector-like quarks. In this case, one would also like to discriminate among the different possible models. For instance, if the only exotic quarks are up quark isosinglets, U^a , then $\alpha_{\phi q}^{(1)} = -\alpha_{\phi q}^{(3)}$ and $X_{ij}^{dL} = \delta_{ij}$, $X_{ij}^{uL} = W_{ik}^L W_{kj}^{L\dagger}$. On the other hand, if they are down quark isosinglets, D^a , $\alpha_{\phi q}^{(1)} = \alpha_{\phi q}^{(3)}$ and $X_{ij}^{uL} = \delta_{ij}$, $X_{ij}^{dL} = W_{ik}^{L\dagger} W_{kj}^L$. These general type of relations between charged and neutral currents are discussed in Ref. [1]. Here we only have to insert the values of the coefficients α_x given in Table 4. In Table 6, we collect the relevant information when only one type of exotic quark gives a sizable contribution.

Remember that these expressions are valid to order $\frac{1}{\Lambda^2}$. We use the following notation in Table 6: $\frac{1}{\Lambda^2}$ indicates that the coupling is $\sim \frac{v^2}{\Lambda^2} \lambda^2$ and that it is given by a positive semidefinite matrix. Similarly, $-\frac{1}{\Lambda^2}$ stands for a negative semidefinite correction, $\pm \frac{1}{\Lambda^2}$ indicates an indefinite correction, and $1 + \frac{1}{\Lambda^2}$ and $1 - \frac{1}{\Lambda^2}$ denote the identity plus a positive and a negative semidefinite contribution, respectively. W^L is the product of a unitary matrix, $V^L = \tilde{V}$, times a hermitian one [1]. The different exotic quark additions can be discriminated by comparing the relations in Table 6 with the experimental couplings of the quarks to the Z , W^\pm and H . These relations are lost in general if more than one kind of vector-like quark is allowed. An exception in which the relations between neutral and charged currents are preserved is the case when the extra quarks belong to two types of multiplets, and these are a singlet and a doublet or a triplet and a doublet. The reason is that singlets and triplets only contribute to left-handed currents, while doublets only contribute to right-handed ones.

On the other hand, as discussed in Ref. [1], these couplings also satisfy certain inequalities. The relevant information comes from the fact that the coefficient matrices α_x in Table 4 are either positive or negative semidefinite. Indeed they are proportional to

Table 6: $Z\bar{q}q'$, $W\bar{q}q'$ and $H\bar{q}q'$ couplings to be measured experimentally (see Eq. (3.1)) and relations they fulfil to order $\frac{1}{\Lambda^2}$ for each type of vector-like quark additions $Q^{(m)}$. In the SM $X_{ij}^{uL} = X_{ij}^{dL} = \delta_{ij}$, $X_{ij}^{uR} = X_{ij}^{dR} = 0$, $W_{ij}^L = V_{ij}$, $W_{ij}^R = 0$, $Y_{ij}^{u,d} = \sqrt{2}\delta_{ij}\frac{m_j^{u,d}}{v}$.

$Q^{(m)}$	X_{ij}^{uL}	X_{ij}^{uR}	X_{ij}^{dL}	X_{ij}^{dR}	W_{ij}^L	W_{ij}^R	$\frac{Y_{ij}^u}{\sqrt{2}}$	$\frac{Y_{ij}^d}{\sqrt{2}}$
U	$W_{ik}^L W_{kj}^{L\dagger}$	0	δ_{ij}	0	$V^L(1 - \frac{1}{\Lambda^2})$	0	$(-\frac{1}{2}\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{uL})\frac{m_j^u}{v}$	$\delta_{ij}\frac{m_j^d}{v}$
D	δ_{ij}	0	$W_{ik}^{L\dagger} W_{kj}^L$	0	$V^L(1 - \frac{1}{\Lambda^2})$	0	$\delta_{ij}\frac{m_j^u}{v}$	$(-\frac{1}{2}\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{dL})\frac{m_j^d}{v}$
$\begin{pmatrix} U \\ D \end{pmatrix}$	δ_{ij}	$\frac{1}{\Lambda^2}$	δ_{ij}	$\frac{1}{\Lambda^2}$	V_{ij}^L	$\pm\frac{1}{\Lambda^2}$	$\frac{m_i^u}{v}(\delta_{ij} - (1 + \frac{1}{2}\delta_{ij})X_{ij}^{uR})$	$\frac{m_i^d}{v}(\delta_{ij} - (1 + \frac{1}{2}\delta_{ij})X_{ij}^{dR})$
$\begin{pmatrix} X \\ U \end{pmatrix}$	δ_{ij}	$-\frac{1}{\Lambda^2}$	δ_{ij}	0	V_{ij}^L	0	$\frac{m_i^u}{v}(\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{uR})$	$\frac{m_i^d}{v}\delta_{ij}$
$\begin{pmatrix} D \\ Y \end{pmatrix}$	δ_{ij}	0	δ_{ij}	$-\frac{1}{\Lambda^2}$	V_{ij}^L	0	$\frac{m_i^u}{v}\delta_{ij}$	$\frac{m_i^d}{v}(\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{dR})$
$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$2\delta_{ij} - W_{ik}^L W_{kj}^{L\dagger}$	0	$-\delta_{ij} + 2W_{ik}^{L\dagger} W_{kj}^L$	0	$V^L(1 + \frac{1}{\Lambda^2})$	0	$(-\frac{1}{2}\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{uL})\frac{m_j^u}{v}$	$(\frac{5}{2}\delta_{ij} - (1 + \frac{1}{2}\delta_{ij})X_{ij}^{dL})\frac{m_j^d}{v}$
$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$	$-\delta_{ij} + 2W_{ik}^L W_{kj}^{L\dagger}$	0	$2\delta_{ij} - W_{ik}^{L\dagger} W_{kj}^L$	0	$V^L(1 + \frac{1}{\Lambda^2})$	0	$(\frac{5}{2}\delta_{ij} - (1 + \frac{1}{2}\delta_{ij})X_{ij}^{uL})\frac{m_j^u}{v}$	$(-\frac{1}{2}\delta_{ij} + (1 + \frac{1}{2}\delta_{ij})X_{ij}^{dL})\frac{m_j^d}{v}$

$\frac{\lambda_{ia}^\dagger \lambda_{aj}'}{M_a^2}$. In the case of the addition of just one type of heavy multiplet, the most stringent bounds for the Z couplings are [1, 10]

$$|X_{ij}|^2 \leq X_{ii}X_{jj}, \quad (4.1)$$

$$|\delta_{ij} - X_{ij}|^2 \leq (1 - X_{ii})(1 - X_{jj}). \quad (4.2)$$

These inequalities are also satisfied for combinations of extra multiplets that preserve the semidefiniteness of the coupling matrices. For X^{uL} (X^{dL}), Eqs. (4.1,4.2) hold for any combination of extra multiplets not containing $Q^{(7)}$ ($Q^{(6)}$). In the case of X^{uR} (X^{dR}), they hold for combinations not containing $Q^{(3)}$ and $Q^{(4)}$ ($Q^{(3)}$ and $Q^{(5)}$) at the same time. Other inequalities are related to the diagonal couplings:

$$X_{ii}^{uL} \leq 1 \quad \text{for combinations without } Q^{(7)}, \quad (4.3)$$

$$X_{ii}^{dL} \leq 1 \quad \text{for combinations without } Q^{(6)}, \quad (4.4)$$

$$X_{ii}^{uL} \geq 1 \quad \text{for } Q^{(7)} \text{ (and doublets)}, \quad (4.5)$$

$$X_{ii}^{dL} \geq 1 \quad \text{for } Q^{(6)} \text{ (and doublets)}. \quad (4.6)$$

The corresponding inequalities for X^R can be read directly from the signs in Table 6.

On the other hand, the W couplings fulfil the relations

$$W_{ik}^L W_{ki}^{L\dagger}, W_{ik}^{L\dagger} W_{ki}^L \leq 1 \quad \text{for combinations without triplets}, \quad (4.7)$$

$$W_{ik}^L W_{ki}^{L\dagger}, W_{ik}^{L\dagger} W_{ki}^L \geq 1 \quad \text{for combinations without singlets}. \quad (4.8)$$

Furthermore,

$$|W_{ij}^R|^2 \leq X_{ii}^{uR} X_{jj}^{dR}. \quad (4.9)$$

Eq. (4.7) ((4.8)) follows from the negative (positive) semidefiniteness of the coefficient matrix $\alpha_{\phi q}^{(3)}$ [1], while Eq. (4.9) is a consequence of the explicit form of the SM correction in Eq. (3.2) and Table 4. The possibility in Eq. (4.8) is not usually emphasized, because isotriplets had not been studied in this context and detail before.

These relations and inequalities may allow to discriminate between different vector-like extensions if new mixing effects are observed. In particular, some clear signatures are:

- Non-zero X^R requires new doublets.
- Non-zero W^R requires new doublets $Q^{(3)}$.
- $X_{ii}^L > 1$ or $(W^L W^{L\dagger})_{ii} > 1$ requires new triplets.
- $(W^L W^{L\dagger})_{ii} < 1$ requires new singlets.

On the other hand, as was shown in Ref. [1], these relations and inequalities can be used to put limits that are independent of the details of the model [10].

Summarizing, we have derived the effective Lagrangian for quark mixing for any SM extension with exotic quarks (Eqs. (3.1), (3.2) and Tables 4 and 5). Due to the particular

form of these corrections, the couplings of two quarks to the Z , W^\pm and H fulfil characteristic relations and inequalities which may allow to discriminate among them and to obtain stringent constraints. In the particular case of the top quark the deviations from the SM predictions must be large to be observable ($\sim 1\%$). We have pointed out that corrections of this size can only arise from vector-like quarks mixing with the SM ones. Therefore, the results in this paper are especially relevant for the physics of top mixing.

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